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PROPAGATION IN TURBULENT ATMOSPHERE

by

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DIRECTION FLUCTUATION OF LASER BEAM  
PROPAGATION IN TURBULENT ATMOSPHERE

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**ABSTRACT:** The formula of arrival angles of spatially partial coherent light beams propagating through turbulent atmosphere is derived from Markovian process approximation, which can be used in both weak and strong intensity fluctuations. We also, have derived the beam wander angle and arrival-angle fluctuation variance of the light beam reflected by a mirror when the beam propagating through the turbulent atmosphere. The reflecting "amplification" characteristics of the beam wander angle and the arrival angles of light beams propagating along a folded path is discussed, which passes the distance twice between the transmitter-receiver and the mirror reflector in weak and strong intensity fluctuations.

1. Introduction

Due to the needs of laser range finding and atmosphere detection with laser radars, researchers have paid growing attention to the statistical properties of various parameters of beams, which twice pass through a randomly fluctuating medium with a particular volume. The theories[1-8] and experiments[2] now available suggest that in the turbulent atmospheres with the same conditions, the fluctuation properties of various parameters of reflected backward waves are different from the statistical properties of optical waves that directly pass through the same path.

In fact, the statistical properties of parameters of backward waves are associated with the geometric shape of reflectors: some reflectors (such as plane mirror) can amplify the deformation of optical waves, while others can otherwise compensate for the deformation of optical waves.

Consequently, it would be useful to carefully study the reflection effects of various reflectors, and to take advantage of these effects in reducing or amplifying the atmospheric turbulence effects, and to increase signal-noise ratio in measuring the atmospheric parameters.

However, up till now, the discussion of reflected beam propagation direction fluctuation has focused on weak intensity fluctuation alone in most cases[1]. Although V.P. Lukin[2] discussed the amplification properties of image trembling in the receiving telescope over a longer propagation distance, he failed to touch upon strong turbulence.

Through the Markov approximation and optical wave composite amplitude normal distribution approximation, this paper discusses the beam arrival-angle fluctuation while directly propagating through the turbulent atmosphere, the wander of reflected backward wave beam at the receiving surface as well as the

statistic properties of the arrival-angle fluctuation.

## 2. Arrival-Angle Fluctuation

The experiments available indicate that the fluctuations in the beam root-mean-square wander angle and arrival angle are both lower than  $10''$  [9]. Based on this, the beam arrival-angle fluctuation problem can be discussed using the paraxial optics approximation. Under the paraxial optics approximation, the random beam arrival angle  $\alpha_0$  at the receiving surface can be derived as [8]

$$\alpha_0 = \frac{d\rho_0}{dx} = \int_0^{\infty} d\xi \iint \nabla_R n_1 I(\xi, \mathbf{R}) t(\mathbf{R}) \frac{d^2 R}{P_0}, \quad (1)$$

where  $n_1$  is refractivity fluctuation;  $R$  is radius vector within the receiving surface:  $t(R) = t_0 \exp(-R^2/\alpha_s^2)$  [10];  $t_0$  is transmittance;  $\alpha_s$  is the radius of the receiving hole;  $I = u(\xi, R) u^*(\xi, R)$  is light intensity within the plane;  $\nabla_R$  is horizontal gradient operator

$$P_0 = \iint \langle I(x, \mathbf{R}) \rangle t(\mathbf{R}) d^2 R,$$

The beam is propagating along the  $x$  axis.

Assume that  $n_1$  is the random quantity of Gaussian distribution and that it satisfies the delta correlation condition, then we can make a systemwide integration averaging of the square of Eq. (1), and apply the Markov approximation [7, 9].

Considering the arrival-angle fluctuation is caused by the optical wave phase fluctuation [2], we can simplify it using the optical wave composite amplitude normal distribution approximation [11]. Then, through computation, we can derive the arrival-angle fluctuation variance of partial coherent beams in the space of the light source

$$\langle \alpha_c^2 \rangle = \frac{4\pi^2 \alpha_{\text{eff}}^4(x)}{\alpha_t^4(x)} \int_0^\infty d\xi \int_0^\infty dK K^2 \phi_n(K) \frac{\alpha_t^2(\xi)}{\alpha_{\text{eff}}^2(\xi)} \left\{ \alpha_t^2(\xi) \exp\left(-\frac{\alpha_t^2(\xi) K^2}{2}\right) \right. \\ \left. + \frac{\alpha_{\text{eff}}^2(\xi)}{[V + \alpha_{\text{eff}}^2(\xi) \alpha_t^{-2}]} \exp\left[-\frac{\alpha_{\text{eff}}^2(\xi) K^2}{2(V + \alpha_{\text{eff}}^2(\xi) \alpha_t^{-2})}\right] \right\}, \quad (2)$$

where

$$\left. \begin{aligned} V(\xi) &= \left[ 3\left(1 - \frac{\xi}{F}\right) + \left(\frac{\xi}{F}\right)^2 + (f^{-2} + \xi^{-1})\xi^2 + \left(1 + \frac{3}{4}\xi^2\right)f^{-2} + \xi^{2-1} \right] \xi^2, \\ \alpha_{\text{eff}}^2(\xi) &= \left[ \frac{1 + \xi^2 + \zeta^2}{f^2} + \left(1 - \frac{\xi}{F}\right)^2 \right] \alpha_0^2, \\ \alpha_t^{-2}(\xi) &= \alpha_{\text{eff}}^{-2}(\xi) + \alpha_t^{-2}, \\ \xi^4 &= \frac{4\alpha_0^2}{\rho_0^2}, \quad \zeta = \frac{\alpha_0}{\rho_0}, \quad f(\xi) = \frac{k\alpha_0^2}{\xi}. \end{aligned} \right\} \quad (3)$$

The  $\phi_n$  in Eq. (2) is three-dimensional turbulent spectrum density;  $K$  is number of waves in space;  $\rho_0$  is the coherence length of partial coherent spherical waves;  $\alpha_t$  is space coherence length of light source;  $\alpha_0$  is the spatial coherence length of light source;  $\alpha_0$  is the equivalent radius of light source;  $k$  is the number of optical waves;  $F$  is the curvature radius of beam. When the light source is a spherical wave or plane wave, Eq. (2) retrogrades to

$$\langle \alpha_c^2 \rangle = 4\pi^2 \int_0^\infty d\xi \int_0^\infty \phi_n(K) K^2 dK \times \left\{ \exp\left(-\frac{\alpha_{\text{eff}}^2(\xi)}{2} K^2\right) + \frac{1}{V} \exp\left(-\frac{\alpha_{\text{eff}}^2(\xi)}{2V} K^2\right) \right\}, \quad (4)$$

where  $\alpha_0 = \alpha_t$ . Eqs. (2) and (4) are the major results in this section.

In weak intensity fluctuation, when the light source is a plane wave and Kolmogorov turbulence spectrum is applied,  $I(\zeta, R) \approx I(0, R)$  [9]. Eq. (4) presents  $\langle \alpha_c^2 \rangle \approx 3 \langle \sigma_c^2 \rangle \approx 5.76 c_n^2 x) 2\alpha_t^{-1/3}$ , which is in agreement with the result obtained by Chiba [4]. Here,  $\langle \sigma_c^2 \rangle$  is beam wander angle variance.

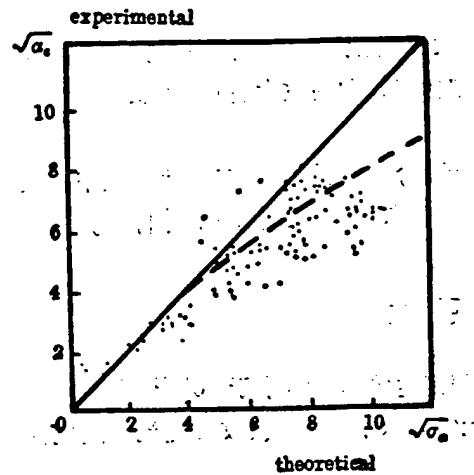


Fig. 1. Comparison of the theory with the experiment

If the Karman spectrum or exponential spectrum[9] is applied in Eqs. (2) and (4), they are also suitable for strong intensity fluctuation, which is confirmed by the agreement between the numerical result\* of Eq. (4) in Fig. 1 (shown by the imaginary line, and Karman spectrum is applied to  $\Phi_0$  and the experiment[5]. The solid line in Fig. 1 corresponds to the slight perturbation approximation theory[5]. The result of Fig. 1 also indicates that the approximation used in deriving Eqs. (2) and (4) is feasible.

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\* Its conditions are:  $(x/F) = -1$ ;  $K_0 = 10.4 \text{ m}^{-1}$ ;  $h_0 = 2 \text{ m}$ ;  $v = 0.3$ ;  $x = 1750 \text{ m}$ ;  
 $\sigma_a^2 = 1.06 c_n^2 x (2\alpha_t)$ ;  $\lambda = 0.5 \mu\text{m}$ .

### 3. Wander Fluctuation of Reflected Beam

When a beam is propagating along a folded optical path of turbulent atmosphere with a plane reflector, whose surface is much larger than the beam cross section, as the reflector, the optical wave composite amplitude  $u$  can meet the following parabolic equation[7]:

$$\left. \begin{aligned} \frac{\partial u}{\partial x} - \frac{i}{2k} \Delta_R u + ikN_1(x, R)u, \\ u(\rho, R) = u_0(R), \end{aligned} \right\} \quad (5)$$

where  $N_1(x, R) = \begin{cases} n_1(x, R), & x < L, \\ n_1(2L-x, R), & L \leq x \leq 2L, \end{cases}$

$\Delta_R$  is Laplace's operator;  $u_0(R)$  is optical wave composite amplitude at the transmission end.

The backward wave composite amplitude  $u_r$  in  $L \leq x \leq 2L$  region can satisfy the following equation:

$$\left. \begin{aligned} \frac{\partial u_r}{\partial x} - \frac{i}{2k} \Delta_R u_r + ikn_1(2L-x, R)u_r, \\ u_r(L, R) = u_i(L, R), \end{aligned} \right\} \quad (6)$$

where  $u_i(L, R)$  is the optical wave composite amplitude incident at the reflecting surface.

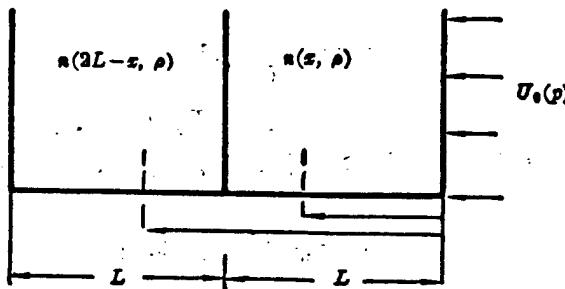


Fig. 2 Light beam propagation path

Fig. 2. Light beam propagation path

Assume that the center of transmitted beam is overlapping with the center of the reflected beam at the receiving end with a transmission optical path as shown in Fig. 2. According to the beam wander theory, let  $\xi = x$ , then in the  $0 \leq \xi \leq L$  region, the vector of the beam "gravity center" position is[8]

$$\rho_o = \frac{1}{P_0} \int_0^L (L - \xi) d\xi \iint d^2 R I(\xi, \mathbf{R}) \nabla_{\mathbf{R}^n} I(\xi, \mathbf{R})_o. \quad (7)$$

While in the  $L \leq \xi \leq 2L$  region, the vector of the beam "gravity center" position is

$$\rho_{cr} = \frac{1}{P'_0} \int_L^{2L} (2L - \xi) d\xi \iint d^2 R I_r(2L - \xi, \mathbf{R}) \nabla_{\mathbf{R}^n} I_r(2L - \xi, \mathbf{R})_o. \quad (8)$$

By ignoring the atmospheric absorption and the backward scattering effect of turbulence,  $P'_0 = P_0$  [2] can be derived through total beam flux conservation. Then, by introducing this relation in Eq. (8), and making variable replacement of  $\xi' = \xi - L$ , Eq. (8) will be converted to

$$\rho_{cr} = \frac{1}{P_0} \int_0^L (L - \xi) d\xi \iint I_r(L - \xi, \mathbf{R}) \nabla_{\mathbf{R}^n} I_r(L - \xi, \mathbf{R}) d^2 R_o. \quad (9)$$

The beam "gravity center" random wander angle at the receiving end can be calculated through superimposition of  $\rho_o/L$  and  $\rho_{cr}/L$  as follows:

$$\begin{aligned} \sigma_{\alpha}(2L) = & \frac{1}{LP_0} \int_0^L (L - \xi) d\xi \iint d^2 R [I(\xi, \mathbf{R}) \nabla_{\mathbf{R}^n} I(\xi, \mathbf{R}) \\ & + I_r(L - \xi, \mathbf{R}) \nabla_{\mathbf{R}^n} I_r(L - \xi, \mathbf{R})]_o. \end{aligned} \quad (10)$$

Similar to the derivation of Eq. (2), by taking advantage of the delta correlation property of  $n_1$  [7], the wander angle fluctuation variance of the reflected beam from the plane reflector can be derived

$$\begin{aligned} \langle \sigma_{\text{ca}}^2(2L) \rangle &= \frac{2\pi L}{P_0^2} \int_0^1 (1-\xi)^2 d\xi \int \int d^2 K K^2 \phi_n(K) \int \int d^2 R_1 d^2 R_2 \exp[iK \cdot (R_1 - R_2)] \\ &\quad \times \langle I(\xi, R_1) I(\xi, R_2) \rangle + \langle \sigma_{\text{cr}}^2(L) \rangle + 2 \langle \sigma_{\text{cs}}^2(L) \rangle, \end{aligned} \quad (11)$$

where the first term is the commonly used single trip beam wander angle fluctuation variance  $\langle \sigma_{\text{c}}^2(L) \rangle$ ;  $\langle \sigma_{\text{cr}}^2(L) \rangle$  is the wander angle fluctuation variance of the reflected backward wave propagating over a distance  $L$ ;  $2 \langle \sigma_{\text{cs}}^2(L) \rangle$  is the contribution of the increased phase fluctuation caused by the interference between the incident light and reflected light to the beam wander. Their specific expressions, respectively, are:

$$\begin{aligned} \langle \sigma_{\text{cr}}^2 \rangle &= \frac{2\pi L}{P_0^2} \int_0^1 (1-\xi)^2 d\xi \int \int \cdots \int d^2 R_1 d^2 R_2 d^2 K K^2 \phi_n(K) \\ &\quad \times \langle I_r(1-\xi, R_1) I_r(1-\xi, R_2) \rangle \exp[iK \cdot (R_1 - R_2)], \end{aligned} \quad (12)$$

$$\begin{aligned} \langle \sigma_{\text{cs}}^2 \rangle &= \frac{2\pi L}{P_0^2} \int_0^1 (1-\xi) \xi d\xi \int \int \cdots \int d^2 R_1 d^2 R_2 d^2 K K^2 \phi_n(K) \\ &\quad \times \langle I(\xi, R_1) I_r(\xi, R_2) \rangle \exp[iK \cdot (R_1 - R_2)]. \end{aligned} \quad (13)$$

Obviously,  $\langle \sigma_{\text{ca}}^2(2L) \rangle \geq \langle \sigma_{\text{c}}^2(2L) \rangle$ . Here,  $\langle \sigma_{\text{c}}^2(2L) \rangle$  is the wander angle fluctuation variance of the beam propagating through the  $2L$  turbulence layer.

In weak Kolmogorov turbulence fluctuation as well as in collimated and focused beam propagation, the  $I(\xi, R) \approx I(0, R) \approx I_r(L-\xi, R)$  approximation can be made [9]. Under this approximation,  $\langle \sigma_{\text{cr}}^2(L) \rangle = 2 \langle \sigma_{\text{cs}}^2(L) \rangle = \langle \sigma_{\text{c}}^2(L) \rangle = [0.099\pi^3 \Gamma(1/6) c_n^2 / 2\sqrt{2}] L (2\alpha_0)^{-1/3} = (1/2) \langle \sigma_{\text{c}}^2(2L) \rangle$  can be derived from Eqs. (12) and (13). By inserting this relation in Eq. (11), the following can be obtained:

$$\langle \sigma_{\text{ca}}^2(2L) \rangle = \frac{3}{2} \langle \sigma_{\text{c}}^2(2L) \rangle. \quad (14)$$

This result conforms to the following result: a finite plane wave, derived through directly and approximately applying the logarithmic normal distribution of optical wave composite amplitude in the wander fluctuation variance and defining  $\langle \rho^2 \rangle = \int \cdots \int d^2 R_1 d^2 R_2 R_1 \cdot R_2 \times \Gamma_u(\rho, R_1, R_2, R_1, R_2) / P_0^2$ , is reflected from the plane reflector and then wander amplified by 3/2 times[1]. However, our conclusion is also applicable to the focused beam propagation.

In strong intensity fluctuation, we now discuss the wander amplification problem based on the analysis of the optical wave mutual coherence function  $\Gamma_u(\rho, R)$  presented in reference[8]. According to the commutation principle, when a beam propagating through a turbulence layer  $x \in (0, L)$  is reflected from a plane reflector located at  $x=L$ , the mutual coherence function of the reflected optical wave is[8]

$$\begin{aligned}
 \Gamma_u(\rho, R) = & \left\langle u_{2L}\left(R + \frac{1}{2}\rho\right) u_{2L}^*(R - \frac{1}{2}\rho) \right\rangle \\
 = & \int \cdots \int u\left(R_0 + \frac{1}{2}\rho_0\right) u^*\left(R_0 - \frac{1}{2}\rho_0\right) \left\langle G\left(0, R_0 + \frac{1}{2}\rho_0; L, R' + \frac{1}{2}\rho'\right) \right. \\
 & \times G^*\left(0, R_0 - \frac{1}{2}\rho_0; L, R' - \frac{1}{2}\rho'\right) G\left(0, R + \frac{1}{2}\rho; L, R' + \frac{1}{2}\rho'\right) \\
 & \left. \times G^*\left(0, R - \frac{1}{2}\rho; L, R' - \frac{1}{2}\rho'\right) \right\rangle d^2 R_0 d^2 \rho_0 d^2 R' d^2 \rho', \quad (15)
 \end{aligned}$$

where  $u(\rho)$  is the optical wave composite amplitude incident at the turbulence layer  $x=0$ ;  $u_{2L}(\rho)$  is the reflected wave composite amplitude at  $x=2L$ ;  $G(0, \rho_0; x, \rho)$  is the spherical wave composite amplitude in the turbulence medium, which can satisfy the following equation:

$$2ik \frac{\partial G}{\partial x} + 4kG = 2k^2 n_1(x, \rho) G, \quad G(0, \rho_0; x, \rho) = \delta(\rho_0 - \rho).$$

Suppose the turbulence fluctuation is sufficiently strong, and after the spherical wave passes through the turbulence layer

with a distance  $L$ , its composite amplitude can meet the normal distribution statistics, then the following can be derived from the statistic property of the normal distribution:

$$\left. \begin{aligned}
 \Gamma_{2L}(\rho, \mathbf{R}) &= \Gamma_{1r}(L, \rho, \mathbf{R}) + \Gamma_{2r}(L, \rho, \mathbf{R}), \\
 \Gamma_{1r}(L, \rho, \mathbf{R}) &= \left( \frac{k}{2\pi L} \right)^2 \int \cdots \int d^2 R_0 d^2 \rho' u \left( \mathbf{R}_0 + \rho' - \frac{1}{2} \rho \right) u^* \left[ \mathbf{R}_0 - \left( \rho' - \frac{1}{2} \rho \right) \right] \\
 &\quad \times \exp \left[ \frac{ik}{L} (\rho - \rho') (\mathbf{R} - \mathbf{R}_0) - D_s(\rho', \rho) - D_s(\rho', 2\rho' - \rho) \right], \\
 \Gamma_{2r}(L, \rho, \mathbf{R}) &= \left( \frac{k}{2\pi L} \right)^2 \int \cdots \int d^2 R_0 d^2 \rho' u \left( \mathbf{R}_0 + \rho' - \frac{1}{2} \rho \right) u^* \left[ \mathbf{R}_0 - \left( \rho' - \frac{1}{2} \rho \right) \right] \\
 &\quad \times \exp \left[ \frac{ik}{L} (\rho - \rho') (\mathbf{R} - \mathbf{R}_0) - D_s(\rho', \mathbf{R}_0 - \mathbf{R} + \rho) \right. \\
 &\quad \left. - D_s(\rho', \mathbf{R} - \mathbf{R}_0 + \rho') \right], 
 \end{aligned} \right\} \quad (16)$$

where  $D_s$  is phase structure function[8];  $\Gamma_{1r}$  is the mutual coherence function of the optical wave that directly passes a distance  $2L$ ;  $\Gamma_{2r}$  is the long-distance correlation function of the reflected field, which is related to the pre-reflection and post-reflection beam propagation through the same inhomogeneous medium. When the turbulence fluctuation is sufficiently strong to make the beam turbulence spread much larger than diffraction spread,  $\langle I_{2r}(L, R) \rangle \approx \langle I_{1r}(L, R) \rangle$ , i.e.  $\langle I_{2L}(L, R) \rangle \approx \langle I_{1r}(L, R) \rangle$  [8] can be derived from Eq. (16). Here,  $\langle I_{1r, 2r}(L, R) \rangle = \Gamma_{1r, 2r}(L, 0, R) \dots$

The foregoing discussion indicates that when turbulence spread is much larger than diffraction spread, the reflection effect of the plane reflector  $\Gamma_{2r}$  is fairly weak, and accordingly, the contribution of  $2\langle \sigma_{cs}^2(L) \rangle$  to  $\langle \sigma_{ca}^2(2L) \rangle$  is fairly small, which will be further explained in the following discussion. In the strong intensity fluctuation, by using optical wave composite amplitude normal distribution approximation, the light intensity mutual correlation function of the reflected field is

$$\langle I_{2L}(\xi, \mathbf{R}_1) I_{2L}(\xi, \mathbf{R}_2) \rangle \approx \langle I_{2L}(\xi, \mathbf{R}_1) \times I_{2L}(\xi, \mathbf{R}_2) \rangle + |\Gamma_{2L}(\xi, \mathbf{R}_1, \mathbf{R}_2)|^2 \approx \langle I_{2L}(\xi, \mathbf{R}_1) \times I_{2L}(\xi, \mathbf{R}_2) \rangle$$

[9]. Since it is known from the foregoing discussion that

$\langle I_{2L}(\xi, \mathbf{R}_1) I_{2L}^2(\xi, \mathbf{R}_2) \rangle \approx \langle I_{2L}(\xi, \mathbf{R}_1) I_{2L}(\xi, \mathbf{R}_2) \rangle$  , then from the wander formula[7]:

$$\langle \sigma_{\alpha}^2(2L) \rangle = 4\pi L P_0^{-2} \int_0^2 (1-\xi)^3 d\xi \iint d^2 K K^2 \phi_n(\mathbf{K}) \int \cdots \int d^2 R_1 d^2 R_2 \\ \times \exp[i\mathbf{K} \cdot (\mathbf{R}_1 - \mathbf{R}_2)] \langle I_{2L}(\xi, \mathbf{R}_1) I_{2L}(\xi, \mathbf{R}_2) \rangle,$$

it is known that  $\langle \sigma_{\alpha}^2(2L) \rangle \approx \langle \sigma_{\alpha}^2(2L) \rangle$ . This suggests that when the ratio between turbulence spread and diffraction spread increases with the increase in the optical wave propagation distance or in turbulence intensity, the wander amplification effect of the plane reflector decreases.

#### 4. Arrival-Angle Fluctuation of Reflected Beam

Based on the above discussion, at the receiving end, the random arrival angle of the beam wavefront reflected from the plane reflector placed in the middle of the optical path can be expressed as

$$\frac{d\rho_{\alpha}}{dx} = \frac{d\rho_{\alpha 1}}{dx} + \frac{d\rho_{\alpha 2}}{dx}, \quad (17)$$

where  $(d\rho_{\alpha 1}/dx)$  is the wavefront fluctuation caused by turbulence in the  $0 \leq \xi \leq L$  region;  $(d\rho_{\alpha 2}/dx)$  is the wavefront fluctuation caused by turbulence in the  $L \leq \xi \leq 2L$  region.

Similar to the derivation of Eq. (2), the reflected beam arrival-angle fluctuation variance can be derived from Eqs. (1) and (17) as follows:

$$\langle \alpha_{\alpha r}^2 \rangle = \left\langle \frac{d\rho_{\alpha 1}}{dx} \cdot \frac{d\rho_{\alpha 2}}{dx} \right\rangle - \langle \alpha_{\alpha}^2(2L) \rangle + 2 \left\langle \frac{d\rho_{\alpha 1}}{dx} \cdot \frac{d\rho_{\alpha 2}}{dx} \right\rangle, \quad (18)$$

where  $\langle \alpha_{\alpha}^2(2L) \rangle$  is the arrival-angle fluctuation variance of the beam directly propagating through the turbulence layer  $2L$ ; the second term of Eq. (18) is the contribution of optical wave interference before and after reflection to the arrival-angle fluctuation, its specified expression being

$$\left\langle \frac{d\rho_{\alpha}}{d\alpha} \cdot \frac{d\rho_{\alpha}}{d\alpha} \right\rangle = \frac{4\pi^2}{P_0^2} \int_0^L d\xi \int_0^L dK K^2 \phi_n(K) \int_{-\infty}^{\infty} \int d^2 R_1 d^2 R_2 \exp[iK \cdot (R_1 - R_2)] \times \langle I(\xi, R) I_r(\xi, R_2) \rangle i(R_1) i(R_2). \quad (19)$$

In weak intensity fluctuation, when a collimated and focused beam serves as the light source, the  $I(\xi, R) \approx I_r(\xi, R) \approx I(0, R)$  approximation can be made. Under this approximation,

$\left\langle \frac{d\rho_{\alpha}}{d\alpha} \cdot \frac{d\rho_{\alpha}}{d\alpha} \right\rangle \approx \langle \alpha_c^2(L) \rangle - (1/2) \langle \alpha_c^2(2L) \rangle$ . From this relation and Eq. (17), the following can be obtained:

$$\langle \alpha_c^2(2L) \rangle = 2 \langle \alpha_c^2(2L) \rangle. \quad (20)$$

This result conforms to the result acquired using other techniques as shown in references [1] and [10]. Similar to the discussion in the previous section, when the turbulence spread is much larger than diffraction spread,  $\langle \alpha_{cr}^2(2L) \rangle$  becomes  $\langle \alpha_c^2(2L) \rangle$ . This shows that not only the wander amplification effect of the plane reflector decreases with the increase in propagation distance, but also the reflected beam arrival-angle fluctuation amplification also decreases when the turbulence intensity  $C_n^2$  increases so that the turbulence spread is much larger than the diffraction spread.

If a reflection kernel function is introduced in the foregoing discussion [1], then it can be known through similar discussion that a triangle prism can reduce the reflected beam wander angle and arrival-angle fluctuation in the weak intensity fluctuation.

## 5. Conclusions

With the "Markov approximation", this paper discussed the arrival-angle fluctuation of partial coherent beams in the turbulent atmosphere, and derived simple expressions applicable for the entire fluctuation. Also, we derived formulas of beam wander angle and arrival-angle fluctuation at a folded path,

which prove to be even more universal compared with the theories available now. Finally, we discussed the reflection amplification property of the beam propagation direction fluctuation in the strong intensity fluctuation, and derived the amplification rate of the wander angle and arrival angle fluctuation of the collimated and focused beam reflected from the plane reflector in the weak intensity fluctuation.

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